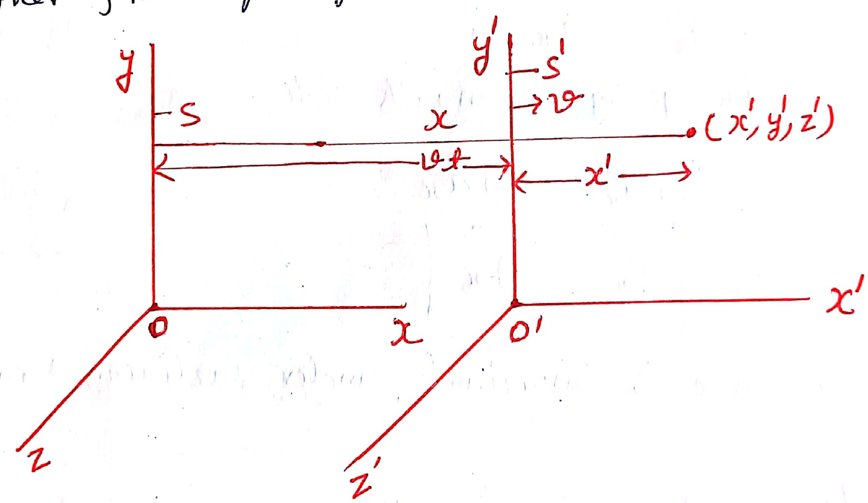
Special theory of relativity

Postulate of special theory of relativity :-

(i). The fundamental laws of physics are same in all inertial frame of reference.

(ii). The speed of light is same in all inertial frame of reference, regardless of the motion of the source relative to the observer.

Galilean Transformation :- It is the set of equations which transform the coordinates of one inertial frame of reference into another frame of reference.



according to the Galilean transformation

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

Inverse Galilean transformation

$$x = x' + vt'$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

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Transformation of Velocity :-

$$x' = x - vt$$

differentiating above equation w.r.to t.

$$\frac{dx'}{dt} = \frac{dx}{dt} - v\frac{dt}{dt}$$

$$\boxed{U_x' = U_x - v}$$

$$U_y' = U_y$$

$$U_z' = U_z$$

$$U_x' = U_x - v$$

$$\frac{dU_x'}{dt} = \frac{dU_x}{dt} - 0 \qquad \left(\frac{dv}{dt} = 0, \because v = \text{~~constant~~ uniform}\right)$$

$$a_x' = a_x$$

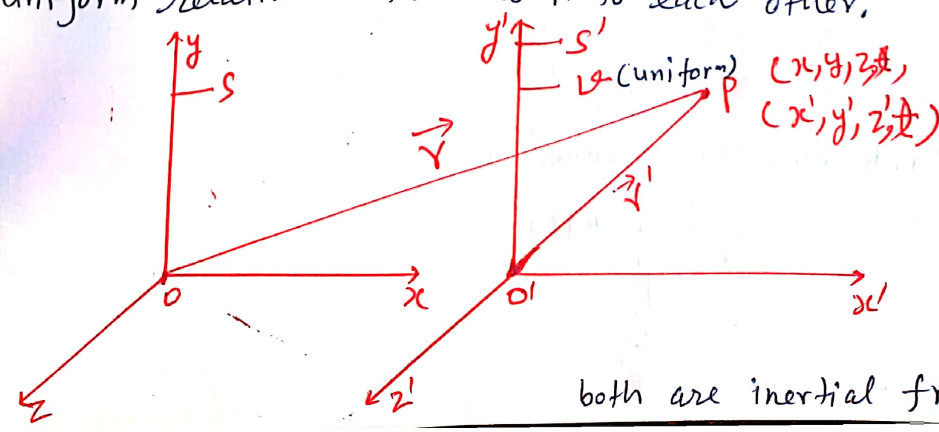
multiply by m on both side.

$$m a_x' = m a_x$$

$$\boxed{F_x' = F_x}$$

Therefore, force is invariant under Galilean transformation.

Lorentz Transformation :- These are the equations which enable us to find the relation between the space and the time co-ordinates of an event in two different inertial frames in uniform relative motion w.r. to each other.



both are inertial frame.

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Consider two frame of reference S and S'. S' is moving with uniform velocity (v). The light wave will appear a sphere of radius 'ct' (c speed of light constant in all frame of reference).

$$OP = ct$$

$$t = \frac{OP}{c} =$$

$$OP = |\vec{r}| = \sqrt{x^2+y^2+z^2}$$

$$OP = (x^2+y^2+z^2)^{1/2}$$

$$t = \frac{(x^2+y^2+z^2)^{1/2}}{c}$$

$$x^2+y^2+z^2 = c^2t^2 \quad \text{—— (I)}$$

Similarly $\quad x'^2+y'^2+z'^2 = c^2t'^2 \quad$ —— (II)

Now from the property of homogenity and isotropy of the free space.

(i) Transformation should be linear and simple.

(ii) at $v <<< c$ these transformation change in Galilean transformation

$$x' = K(x-vt)$$

inverse transformation

$$x = K(x'+vt')$$

$$x = K[K(x-vt)+vt']$$

$$x = K^2(x-vt)+Kvt'$$

$$x = K^2x - K^2vt + Kvt'$$

$$Kvt' = x - K^2x + K^2vt$$

$$Kvt' = K^2vt + (1-K^2)x$$

$$t' = \frac{\cancel{K^2}\cancel{v}t}{\cancel{K}\cancel{v}} + \frac{x}{Kv}(1-K^2)$$

$$t' = Kt + \frac{x}{Kv}K^2\left(\frac{1}{K^2}-1\right)$$

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$$t' = K\left[t + \frac{x}{v}\left(\frac{1}{K^2} - 1\right)\right] \quad \text{----} \quad (iii)$$

c is same in all frame then

$$x = ct \quad , \quad x' = ct'$$

$$x' = ct'$$

$$K(x - vt) = cK\left[t + \frac{x}{v}\left(\frac{1}{K^2} - 1\right)\right]$$

$$K(ct - vt) = cKt + cK\frac{x}{v}\left(\frac{1}{K^2} - 1\right)$$

$$\cancel{Kct} + Kvt = \cancel{cKt} + cK\frac{x}{v}\left(\frac{1}{K^2} - 1\right)$$

$$-Kvt = \frac{cx}{vK} - \frac{cKx}{v}$$

$$-Kv^2t = \frac{cx}{K} - cKx$$

$$-Kv^2t + cKx = \frac{cx}{K}$$

$$-v^2t + c \times ct = \frac{c \times ct}{K^2}$$

$$\cancel{ct}\left(-\frac{v^2}{c} + c\right) = \frac{c \times \cancel{ct}}{K^2}$$

$$\cancel{c}\left(1 - \frac{v^2}{c^2}\right) = \frac{\cancel{c}}{K^2}$$

$$\left(1 - \frac{v^2}{c^2}\right) = \frac{1}{K^2} \Rightarrow K = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$t' = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\left[t + \frac{x}{v}\left(\frac{\cancel{1}}{(1 - v^2/c^2)} - 1\right)\right]$$

$$t' = \frac{1}{\sqrt{1 - v^2/c^2}}\left[t + \frac{x\, v^2/c^2}{\cancel{v}(\cancel{\;})}\right]$$

$$t' = \frac{1}{\sqrt{1 - v^2/c^2}}\left[t - \frac{vx}{c^2}\right]$$

$$\boxed{t' = \frac{1}{\sqrt{1 - v^2/c^2}}\left[t - \frac{vx}{c^2}\right]}$$

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Lorentz transformation are.

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$$

Lorentz inverse transformation.

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - v^2/c^2}}$$

if $v <<< c$ then $v^2/c^2 << 1$ so they change into Galilean transformation.

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